Chapter-3 Calculus

- Integral and differential Calculus (series < Taylor, Mclaurin)
- Vector Calculus — differential, Integral
- Maxima & Minima
- Mean Value theorem

Properties of Definite Integral :

i) $\int_a^b f(x)\,dx = [\phi(x)]_a^b = \phi(b) - \phi(a)$

ii) $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

iii) $\int_a^b f(x)\,dx = \int_a^b f(y)\,dy$

iv) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

v) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

vi) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

vii) $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(-x) = f(x) \text{ even} \\ 0 & ; \ f(-x) = -f(x) \text{ odd function} \end{cases}$

Scanned by CamScanner

viii) $$\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^{a} f(x) & : f(2a-x) = f(x) \\ 0 & : f(2a-x) = -f(x) \end{cases}$$

eg① $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx$ is

a) 0
b) $a/2$
c) a
d) $2a$

eg② $\int_0^{\pi/4} \frac{1-\tan x}{1+\tan x}\,dx$ is a) 0 b) 1 c) $\ln 2$ d) $2\ln 2$

eg③ $\int_0^{\pi/2} \frac{\cos x + i\sin x}{\cos x - i\sin x}\,dx$ is

a) 0
b) -1
c) i
d) $2i$

Sol:) $I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}\,dx$ $\qquad I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx$

$\Rightarrow$ ~~$\int_0^a \sqrt{a-x}\,(\sqrt{a-x}$~~

$$I + I = \int_0^a \frac{\sqrt{a-x}+\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}\,dx$$

$$I = a/2 \;//$$

Scanned by CamScanner

Sol 2 $I = \int_0^{\pi/4} \frac{1-\tan x}{1+\tan x}\, dx$

$I = \int_0^{\pi/4} \frac{\cos^2 x + \sin^2 x - 2\sin x \cos x}{\cos 2x}$

$= \int_0^{\pi/4} \frac{1 - \sin 2x}{\cos 2x}$

$= \int_0^{\pi/4} \sec 2x - \int_0^{\pi/4} \tan 2x$

$\left[\frac{1}{2}\log(\sec 2x)\right]_0^{\pi/4} = \frac{1}{2}\ln 2$

Q3. $\int_0^{\pi/2} \frac{e^{ix}}{e^{-ix}}\, dx$

$\int_0^{\pi/2} e^{2ix}\, dx$

$\left(\frac{e^{2ix}}{2i}\right)_0^{\pi/2} = \frac{e^{i\pi}}{2i} - \frac{1}{2i}$

$= \frac{e^{i\pi} - 1}{2i} = \frac{\cos\pi + i\sin\pi - 1}{2i}$

$= \frac{-2}{2i} = i$

$= c$ //

Scanned by CamScanner

Q. $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$ a) 0 B) $\pi/2$ c) $\pi/4$ d) Not

Sol $\pi/4$ { half of upper limit for such problem }

Q5, 15, 21, 27, 43, 44, 48, 49

Sol 5 : $\int_0^{\pi} \sin^3\theta\, d\theta$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\Rightarrow$ ~~Sin π~~ $\cdot \int_0^{\pi} \frac{3\sin\theta - \sin 3\theta}{4}\, d\theta$

$\Rightarrow -\frac{3}{4}(\cos\theta)_0^{\pi} + \frac{1}{4}\left(\frac{\cos 3\theta}{3}\right)_0^{\pi}$

$\Rightarrow -\frac{3}{4}[-1-1] + \frac{1}{12}(\cos\pi - 1)$

$= +\frac{3}{2} - \frac{1}{6}$

$\frac{9-1}{6} = 8/6$

$= 4/3$ //

or $\int_0^{\pi} (1-\cos^2\theta)\sin\theta\, d\theta$

$\Rightarrow$

Q 15

Sol $\int_0^{2} \frac{(x-1)^2 \sin(x-1)}{(x-1)^2 + \cos(x-1)}$

~~f(2a+1)~~ $f(2a-x)$

$= ~~f(2~~ f(2-x)$

$(2-x-1)^2$

$(1-x)$

$-(\sin(1-x))$

$\Rightarrow$

0 // B)

Scanned by CamScanner

Q21 $\left(\tan^{-1}x\right)_{-\infty}^{\infty}$

$\Rightarrow \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi$ //

Q27. $\int_{-a}^{a} \left(\sin^6 x + \sin^7 x\right) dx$

$\Rightarrow 2\int_0^a \sin^6 x$ //.

Q43, 44. $I = \int_0^{\pi/4} \cos^2 x \, dx$

$\Rightarrow \frac{1+\cos 2x}{2}$

$\frac{1}{2}\left(\frac{\pi}{4}\right) + -\left(\frac{\sin 2x}{4}\right)_0^{\pi/4}$

$\frac{\pi}{8} + \frac{1}{4}$ // A)

Q48. $\int \log x \, dx$

Sol ~~$\frac{1}{x} \cdot x - \log x$~~ ~~log~~ ~~log x~~

$\int 1 \cdot \log x \, dx$

$\Rightarrow \log x \int dx - \int \frac{d}{dx} \log x \int dx$

$\Rightarrow x \log x - \int dx$

$\Rightarrow x \log x - x$

$= x(\log x - 1)$ //.

Scanned by CamScanner

* Special function for Single Integral.

1) Gamma function :-

$$\Gamma(n) = \int_0^\infty \underbrace{e^{-x}}_{\text{must } e^{-x}} \cdot x^{n-1}\, dx$$

for $n=1$ $\quad \Gamma(1) = \int_0^\infty e^{-x}\, dx = -\left[e^{-x}\right]_0^\infty$

$\Gamma(1) = 1$

- $\Gamma(1) = 1$
- $\Gamma(n+1) = n\Gamma(n)$ $\quad n =$ fraction
- $\Gamma(1/2) = \sqrt{\pi}$
- $\Gamma(n+1) = n!$ $\quad (n =$ Integer
- $\Gamma(-1/2) = -2\sqrt{\pi}$

$$\left[\begin{array}{l} \Gamma\left(\frac{-1}{2}+1\right) = \sqrt{\pi} \\ -\frac{1}{2}\Gamma\left(\frac{-1}{2}\right) = \sqrt{\pi} \\ \Gamma\left(\frac{-1}{2}\right) = -2\sqrt{\pi} \end{array}\right.$$

eg① $I = \int_0^\infty e^{-x^2/2}\, dx$

$x^2/2 = t$

$dx = \frac{dt}{\sqrt{t}\sqrt{2}}$ $\quad x=0,\ t=0$ $\quad x=\infty,\ t=\infty$

$$I = \int_0^\infty e^{-t} \frac{dt}{\sqrt{t}\sqrt{2}}$$

$$I = \int_0^\infty \frac{e^{-t}\, t^{-1/2}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\Gamma(1/2) = \sqrt{\pi/2}\ //.$$

Scanned by CamScanner

Trigonometric form

$$\int_0^{\pi/2} \sin^p\theta \cos^q\theta = \frac{\Gamma\frac{p+1}{2}\,\Gamma\frac{q+1}{2}}{2\,\Gamma\frac{p+q+2}{2}}$$

Q1. The Value of $\int_0^a \frac{x^7}{\sqrt{a^2-x^2}}\,dx$

A) $8/35\ a^7$
B) $16/35\ a^7$
C) $32/35\ a^7$
D) $64/35\ a^7$

Q2. $\int_{-\infty}^{0} e^{-x^2/20}\,dx$

a) $1/2$
b) $\sqrt{10}$
c) $\sqrt{5\pi}$
d) $\sqrt{\pi}$

Q3 $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2}\,dx$

a) $1/2$
b) 1
c) $\sqrt{\pi}$
d) 2π

Gaussian function / even function

Sol 1 ~~$a^2 - x^2$~~ $x = a\sin\theta$

$x^2 = a^2\sin^2\theta$

$$\int_0^{a\sin\theta} \frac{x^6 \cdot x}{\sqrt{a^2 - a^2\sin^2\theta}} = \int \frac{a^2\sin^2\theta \cdot a\sin\theta}{a\sqrt{1-\sin^2\theta}}$$

$$= a^{12}\int \frac{\sin^{13}\theta}{\cos\theta}$$

$$a^{12}\int_0^{\pi/2} \sin^{13}\theta \cos^{-1}\theta$$

$$\frac{a^{12}\,\Gamma 7\,\Gamma 0}{2\,\Gamma 7} \Rightarrow \frac{a^{12}}{2\,\Gamma 6+1}$$

Scanned by CamScanner

$$\Rightarrow \int \frac{x^6 \cdot x}{\sqrt{a^2 - x^2}}\, dx$$

Sol $x = a\sin\theta \qquad a\cos\theta\, d\theta = dx$

$$\int_0^{\pi/2} \frac{a\sin\theta \cdot a^6 \sin^6\theta}{a\sqrt{1-\sin^2\theta}}\, d\theta \cdot a\cos\theta$$

$$\Rightarrow \int_0^{\pi/2} a^7 \sin^7\theta$$

$$\Rightarrow \frac{a^7 \lfloor 7+1}{2 \lfloor \frac{8}{2}} = \frac{a^7 \lfloor 6+1}{2 \lfloor 3+1} \Rightarrow \frac{a^7\, 6\times5\times4\times3!}{2\, 3!} \Rightarrow$$

$$\Rightarrow \frac{a^7 \lfloor \frac{7+1}{2} \lfloor \frac{1}{2}}{2 \lfloor \frac{7+0+2}{2}} = \frac{a^7 \lfloor 4 \lfloor 1/2}{2 \lfloor \frac{9}{2}}$$

$$= \frac{a^7 \lfloor 3+1 \sqrt{\pi}}{2 \cdot \frac{7}{2}\times\frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}\times\sqrt{\pi}}$$

$$= a^7 \times 16/35 \;//.$$

Q2. Sol $\int_{-\infty}^{0} e^{-x^2/20}\, dx$ even function

$$= \int_0^{\infty} e^{-x^2/20}\, dx = \int_0^{\infty} e^{-t} \frac{10\, dt}{\sqrt{20t}}$$

$x^2 = 20t \qquad \frac{x^2}{20} = t \qquad 2x\,dx = 20\,dt$

$x = \sqrt{20t} \quad 1/\sqrt{20}$

$$= \int_0^{\infty} t^{\frac{1}{2}} e^{-t} \frac{10}{\sqrt{20}}$$

Scanned by CamScanner

$\frac{10}{\sqrt{20}}\sqrt{1/2}$ $\quad\quad$ $\frac{\sqrt{20}}{10}\sqrt{1/2}$

$\Rightarrow \frac{10}{\sqrt{20}}(-2\sqrt{\pi}) = \sqrt{5\pi}$ //.

$\Rightarrow \frac{10}{\sqrt{5\times2\times2}}(-2\sqrt{\pi})$

$\Rightarrow -10\sqrt{\pi}$

* **Beta function :**

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx \quad \text{---} \quad (1)$$

$(1-x)^{n-1}$ must convert to $(1-x)$

Let:

$1-x = y \Rightarrow -dx = dy$

$x \to 0 \Rightarrow y \to 1$

$x \to 1 \Rightarrow y \to 0$

$$\beta(m,n) = \int_1^0 (1-y)^{m-1}(y)^{n-1}(-dy)$$

$$= \int_0^1 y^{n-1}(1-y)^{m-1}\,dy$$

$\boxed{B(m,n) = \beta(n,m)}$ duality of β functio

Scanned by CamScanner

Trigonometric form of β function

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

$x = \sin^2\theta \Rightarrow dx = 2\sin\theta\cos\theta\,d\theta$

$x \to 0 \quad \theta \to 0$

$x \to 1 \quad \theta \to \pi/2$

$$\beta(m,n) = \int_0^{\pi/2} (\sin^2\theta)^{m-1}(\cos^2\theta)^{n-1}\,2\cos\theta\sin\theta\,d\theta$$

$$\boxed{\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta\cos^{2n-1}\theta\,d\theta}$$

Gamma :-

$$\int_0^{\pi/2} \sin^p\theta\cos^q\theta\,d\theta = \frac{\sqrt{\frac{p+1}{2}}\sqrt{\frac{q+1}{2}}}{2\sqrt{\frac{p+q+2}{2}}}$$

$$\beta(m,n) = 2 \times \frac{1}{2}\frac{\sqrt{m}\sqrt{n}}{\sqrt{m+n}}$$

$$\boxed{\beta(m,n) = \frac{\sqrt{m}\sqrt{n}}{\sqrt{m+n}}}$$

Scanned by CamScanner

Q1. $\int_0^4 x^4 (1-\sqrt{x})^5 \, dx$

a) $\frac{1}{1515}$ b) $\frac{1}{15015}$ c) $\frac{1}{5015}$ d) $\frac{1}{1015}$

Sol) $\sqrt{x} = t$ $\quad x = t^2$

$\frac{1}{2} x^{1/2} = t$

$\frac{1}{2} x^{-1/2} dx = dt$

$dx = \frac{2dt\sqrt{x}}{}$

$= 2t\,dt$

$\int_0^4 t^8 (1-t)^5 \, 2t \, dt$

$2\int_0^4 t^9 (1-t)^5 \, dt$

$m = 10$

$n = 6$

$= 2\frac{\lfloor 10 \lfloor 6}{\lfloor 16}$

$= 2\frac{\lfloor 9+1 \lfloor 5+1}{\lfloor 15+1}$ $= \frac{2 \times 9! \times 5!}{15!}$

$= \frac{2 \times 9! \times 5!}{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9!}$

$= \frac{2 \times 5 \times 4 \times 3 \times 2}{15 \times 7 \times 13 \times 11}$

$\frac{1}{15 \times 11 \times 13 \times 7}$

$= \frac{1}{15015}$ //

Scanned by CamScanner

Q2
Sol $\int_0^1 (1-x^2)^{3/2} x^4 \, dx$

$x^2 = t$

$2x\,dx = dt$

$dx = \frac{dt}{2\sqrt{t}}$

$\int_0^1 \frac{(1-t)^{3/2} \cdot t^2}{2t^{1/2}}$

$\frac{5}{2} - 1$

$2 - \frac{1}{2}$

$\int_0^1 \frac{(1-t)^{\frac{5}{2}-1} \cdot t^{3/2}}{2}$

$\frac{1}{2} - 1$

$\frac{1}{2} \int (1-t)^{5/2-1} \, t^{5/2-1}$

$\frac{1}{2} \frac{\lceil \frac{5}{2} \lceil \frac{5}{2}}{\lceil 5} = \frac{\frac{1}{2} \times \frac{3}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{3}{2} \times \frac{1}{2} \times \pi}{4 \times 3 \times 2}$

$= \frac{25 \times 3\pi}{2^8} \quad \frac{3\pi}{2^8} = \frac{3\pi}{256}$ //

Scanned by CamScanner

* Double and triple Integral (Multiple Integral)

Double Integral

a) Constant limit

$$I = \int_{a}^{b} \int_{c}^{d} f(x,y)\, dx\, dy$$

b) Variable limit

a) $I = \int_{1}^{2} \int_{x}^{\sqrt{x}} f(x,y)\, dy\, dx$

b) $\int_{0}^{9} \int_{y^2}^{y} f(x,y)\, dx\, dy$

c) $I = \int_{0}^{b} \int_{x}^{\sqrt{x}} \int_{x-y}^{x+y} f(x,y,z)\, dz\, dy\, dx$

$$\int_{0}^{1} \int_{0}^{2} \int_{0}^{3} f(x,y,z)\, dx\, dy\, dz$$

Order.

z y x

Scanned by CamScanner

Q4:, 16, 25, 30, 51

Sol 4 $I = \int_1^2 y\,dx$

$I = \int_1^2 (x+1)\,dx$

$= \left(\frac{x^2}{2} + x\right)_1^2$

$= \frac{4}{2} + 2 - \frac{1}{2} - 1$

$= 4 - \frac{3}{2}$

$= \frac{8-3}{2} = 5/2 = 2.5$

Q16.

Sol $\int_0^2 \int_0^x e^{x+y}\,dy\,dx$

$\Rightarrow \int_0^2 \left(\int_0^x e^x \cdot e^y\,dy\right) dx$

$\Rightarrow \int_0^2 \left[e^x (e^y)_0^x\right] dx$

$\int_0^2 e^x (e^x - 1)\,dx$

$\int_0^2 e^x e^x\,dx - \int_0^2 e^x\,dx$

$\frac{(e^4 - 1)}{2} - (e^2 - 1)$

$\frac{e^4}{2} - \frac{1}{2} + 1 - e^2$

$\Rightarrow \frac{-e^2}{2}$

$e^4 - 2e^2 + 1$

$\int_0^2 (e^{2x} - e^x)\,dx$

$\Rightarrow \int_0^2 e^{2x} - \int_0^2 e^x\,dx$

$\Rightarrow \left(\frac{e^{2x}}{2}\right)_0^2 - (e^x)_0^2$

$= \frac{e^4}{2} - \frac{1}{2} - e^2$

$= \frac{e^4}{2} - e^2 - \frac{1}{2} =$

$\frac{1}{2}\left[e^4 - e^2\right]$

Scanned by CamScanner

Q30 $V = \int_0^{2\pi}\int_0^{\pi/3}\int_0^1 r^2 \sin\phi \, dr\, d\phi\, d\theta$,

$$\int_0^{2\pi}\int_0^{\pi/3} \sin\phi \, \frac{r^3}{3} \, d\phi\, d\theta$$

$$\Rightarrow \int_0^{2\pi} -\frac{r^3}{3}(\cos\phi)_0^{\pi/3} \, d\theta$$

$$= -\int_0^{2\pi} \frac{1}{3}\,\frac{1}{2}\, d\theta$$

$$= +\frac{1}{6}(2\pi)$$

$$= 2\pi/3 \quad //$$

Q. $I = \int_0^1 \int_{9y}^3 e^{x^2}\, dx\, dy$ → Tough

a) $1/6(e^9 - 1)$
b) $1/9(e^6 - 1)$
c) $1/6(e^9 + 1)$
d) $1/9(e^6 + 1)$

$x^2 = t$

$2x\,dx = dt$

$dx = \frac{dt}{2\sqrt{t}}$

$$\Rightarrow I = \int_0^1 \int_{9y^2}^9 e^t \frac{dt}{2\sqrt{t}}\, dy$$

~~$I = \frac{1}{2}\int_0^1$~~

* Change of order of Double Integral :-

I) Identify the limits.

II) Trace the limit.

III) Identify the stripe.

IV) By changing the horizontal strip into vertical or vice versa we can get the changed order

$x_{low} = 3y$

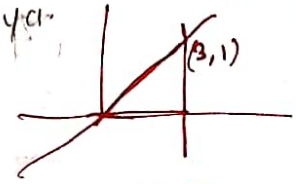
$x_{up} = 3$

line parallel to y axis

line pass through (0,0)

$y = 0$ x axis

$y = 1$ line pass to x axis

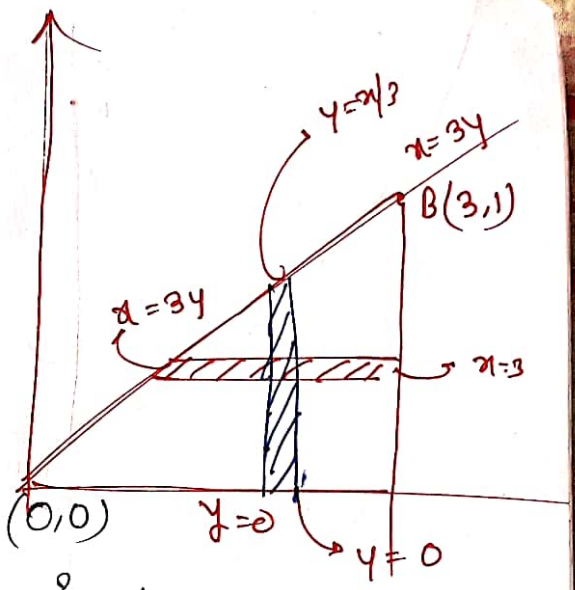


Changed order

$$I = \int_0^3 \int_0^{x/3} e^{x^2}\, dy\, dx$$

$$= \int_0^3 e^{x^2}\left(\frac{x}{3} - 0\right) dx$$

$$= \frac{1}{3}\int_0^3 x e^{x^2}\, dx$$



$$\Rightarrow \frac{1}{3}\int_0^3 e^t \frac{dt}{2}$$

$x^2 = t$

$2x\,dx = dt$

$x\,dx = dt$

$$\frac{1}{6}\left(e^t\right)_0^3 \quad \frac{1}{2}\left[\frac{e^9}{3} - \frac{1}{3}\right] = \frac{1}{6}\left(e^9 - 1\right) \quad //$$

Q2. $I = \int_0^1 \int_{x^2}^{2-x} f(x,y)\, dy\, dx$

$\Rightarrow$ x $\quad$ $y \rightarrow$ verticle

Limits

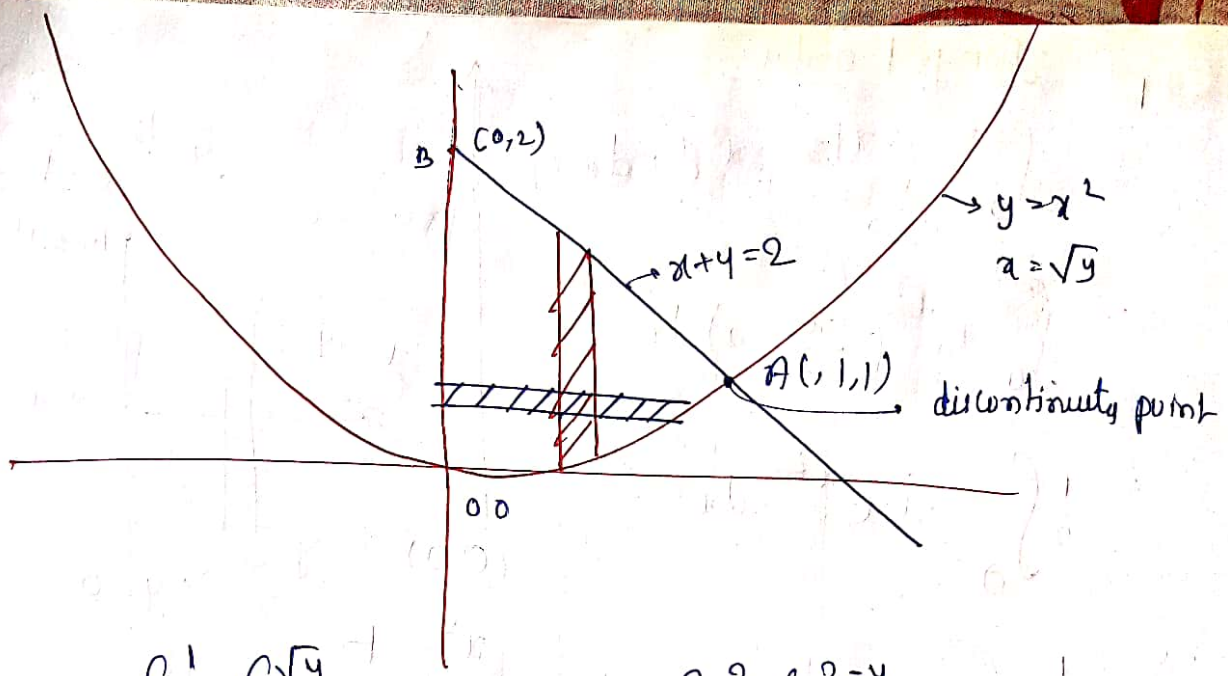
$y = x^2$ (parabola)

$y = 2 - x \Rightarrow x + y = 2$ (line)

$x = 0$ (y axis)

$x = 1$ (line $\parallel$ to y axis)

Scanned by CamScanner



$$\int_0^1 \int_0^{\sqrt{y}} f(x,y)\,dx\,dy + \int_1^2 \int_0^{2-y} f(x,y)\,dx\,dy$$

Q25; 28, 26

Q25

Sol)

$$\int\int xy\,dy\,dx$$

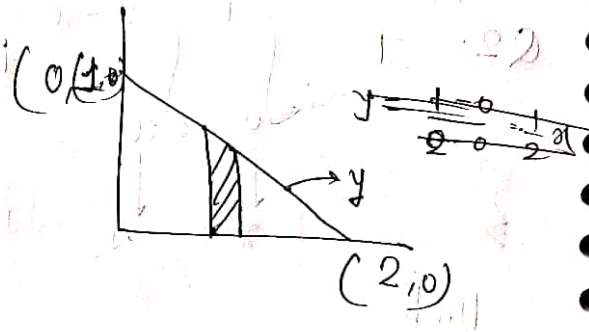
$\Rightarrow$

$$\int_0^2 \int_0^{1-x/2} xy\,dy\,dx$$

$$\int_0^2 x\left(\frac{y^2}{2}\right)_0^{1-x/2} dx$$

$$\frac{1}{2}\int_0^2 x\left(\left(1-\frac{x}{2}\right)^2 - 0\right)$$

$$\frac{1}{2}\int_0^2 x\left(1+\frac{x^2}{4}-x\right)$$



$y = mx + c$

$y = \frac{0-1}{2-0}x + 1$

$y = -\frac{1}{2}x + 1$

$\Rightarrow \left(\frac{x^2}{2}\right)_0^2 + \left(\frac{xy}{16}\right)_0^2 - \left(\frac{x^3}{3}\right)_0^2$

$2 + 1 - \frac{8}{3} = 1/6$ //

Scanned by CamScanner

$$\frac{1}{2}\left[\int_0^2 \left(x + \frac{x^3}{4} - x^2\right) dx\right.$$

$$\Rightarrow \frac{1}{2}\left[\left[\frac{x^2}{2}\right]_0^2 + \left(\frac{x^4}{16}\right)_0^2 - \left(\frac{x^3}{3}\right)_0^2\right]$$

$$\Rightarrow \frac{2}{6} + \frac{1}{1} - \frac{8}{9} = 1/6$$

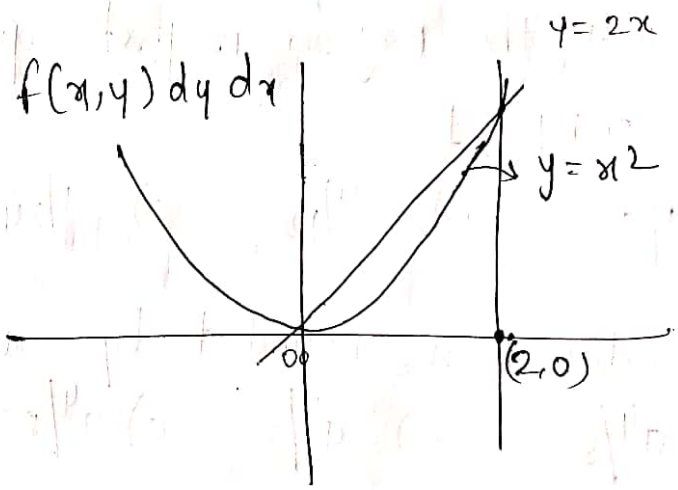
$$\Rightarrow \frac{27 - 8}{9}$$

$$\Rightarrow \frac{+6}{9}$$

$$\Rightarrow$$

Q46

Sol $\int_0^2 \int_{x^2}^{2x} f(x,y)\, dy\, dx$



Limit

$y = x2$

$y = 2x$

$x = 0$

$x = 2$

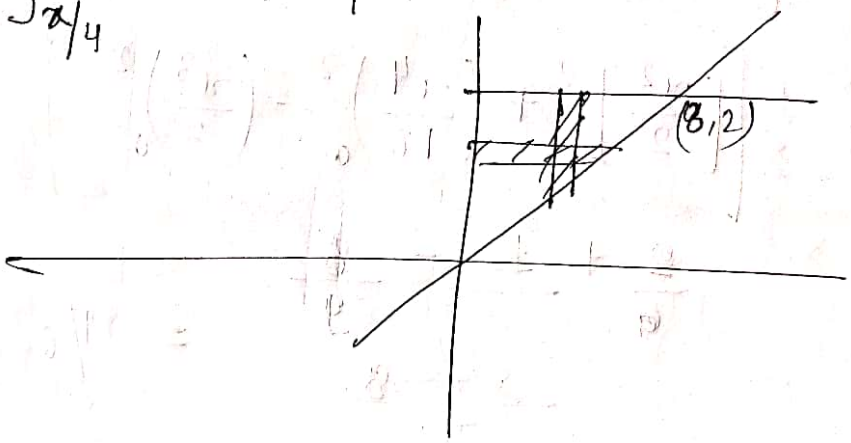
Scanned by CamScanner

Q28

Sol

$$I = \int_0^8 \int_{x/4}^{2} f(x,y)\, dy\, dx$$

$x = 4y$



Q. Find the Value of Integral

$$I = \iint_R xy\, dx\, dy$$ where R is

① R is the Region in first quadrant and $x + y \leq 1$

a) 1/8 b) 1/12 c) 1/24 d) 1/62

② where R is bounded by $x = 2a$ & $x^2 = 4ay$

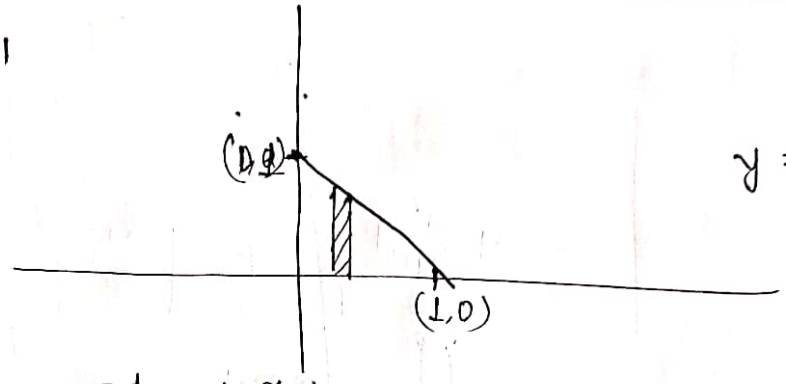
a) $a^4/2$ b) $a^4/3$ c) $a^4/5$ d) a^4

③ If W R is a quadrant of circle $x^2 + y^2 = a^2$ where $x \geq 0, y \geq 0$

a) $a^4/8$ b) $a^4/16$ c) $a^4/24$ d) $a^4/32$

Scanned by CamScanner

Sol:



$y = 1 - x$

$$\int_0^1 \int_0^{1-x} xy \, dx \, dy$$

$$\Rightarrow \int_0^1 y \left(\frac{x^2}{2}\right)_0^{1-x} dy \Rightarrow \frac{1}{2}\left(\frac{y^2}{2}\right)_0^1$$

$$\frac{1}{2} \cdot \frac{1}{1}$$

$$\Rightarrow \int_0^1 \int_0^{1-x} xy \, dy \, dx$$

$$\Rightarrow \int_0^1 x \left(\frac{y^2}{2}\right)_0^{1-x} dx$$

$$\Rightarrow \int_0^1 x \left(\frac{(1-x)^2}{4}\right) dx$$

$$\int_0^1 x\left(\frac{1 + x^2 - 2x}{4}\right) dx$$

$$\int_0^1 \frac{x + x^3 - 2x^2}{4}$$

$$\frac{1}{4}\left[\frac{x^2}{2} + \frac{x^4}{4} - \frac{2x^3}{3}\right]_0^1$$

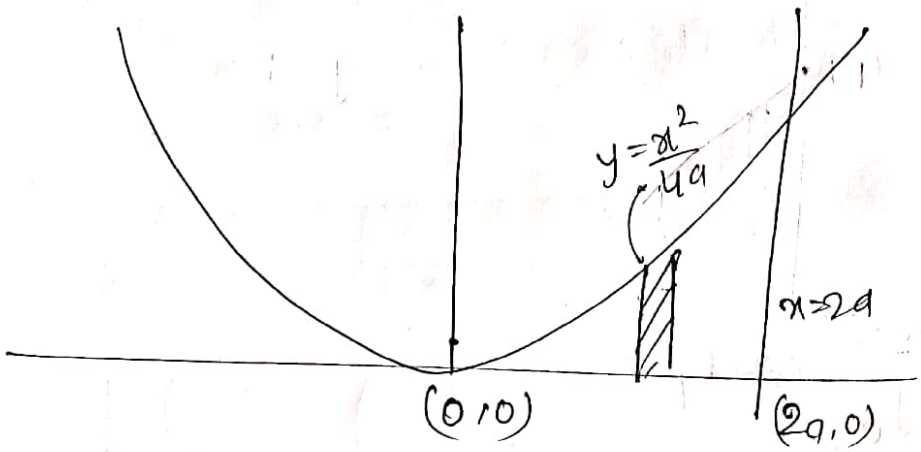
$$\frac{1}{4}\left[\frac{1}{2} + \frac{1}{4} - \frac{2}{3}\right] = \frac{12 + 6 - 16}{24}\left(\frac{2}{24}\right)^{\frac{1}{2}}$$

$$\Rightarrow 1/24$$

Scanned by CamScanner

(2)

Sol



$x = 2a, \; x^2 = 4ay$

$$I = \int_0^{2a} \int_0^{x^2/4a} xy \, dy \, dx$$

$$= \int_0^{2a} x \left(\frac{y^2}{2}\right)_0^{x^2/4a} dx$$

$$\Rightarrow \int_0^{2a} x \left[\frac{x^4}{2 \times 16a^2}\right] dx$$

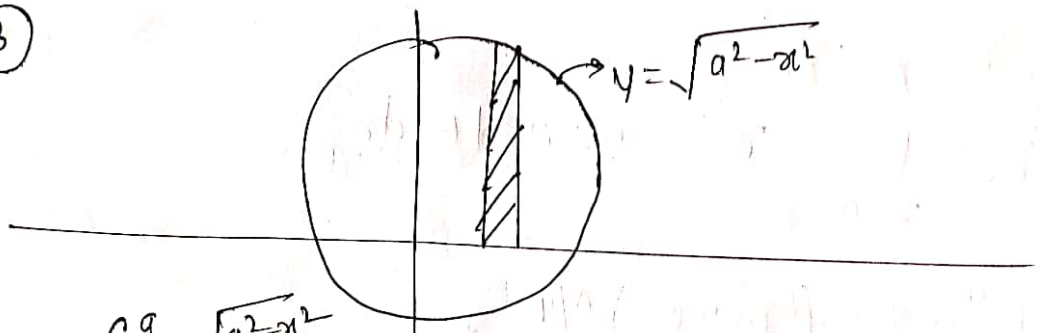
$$\Rightarrow \left[\frac{x^6}{6}\right]_0^{2a} \frac{1}{32a^2} = \frac{2^6 a^6}{6} \frac{1}{32a^2}$$

$$= \frac{2}{6} a^4$$

$$= a^4/3 \; //$$

Scanned by CamScanner

(3)



$$I = \int_0^a \int_0^{\sqrt{a^2-x^2}} (xy)\, dy\, dx$$

$$\Rightarrow \int_0^a x \left[\frac{y^2}{2}\right]_0^{\sqrt{a^2-x^2}} dx$$

$$\int_0^a x \left(\frac{a^2-x^2}{2}\right) dx$$

$$\int_0^a \frac{a^2}{2}x - \frac{x^3}{2}$$

$$\frac{a^2}{2}\left(\frac{x^2}{2}\right)_0^a - \left(\frac{x^4}{8}\right)_0^a$$

$$\Rightarrow \frac{a^2}{4}\cdot a^2 - \frac{a^4}{8}$$

$$\Rightarrow \frac{a^4}{4} - \frac{a^4}{8}$$

$$\frac{2a^4 - a^4}{8} = a^4/8 \;//.$$

Scanned by CamScanner

Q. $\frac{1}{2}\int_0^a \int_0^{\pi/4} r^3 \sin 2\theta \, d\theta \, dr$

$\Rightarrow \frac{1}{2}\int_0^a r^3 \left(\frac{\cos 2\theta}{2}\right)_0^{\pi/4} dr$

$-\frac{1}{2}\int_0^a r^3 \left(-\frac{1}{2}\right) dr$

$\frac{1}{4} \quad \frac{r^4}{4} = \frac{a^4}{16}$ //.

*** Note: For Surface Integration or double Integration of circle It is easier to solve in polar form as Compared to Cartesian because limits are Converted into Constant

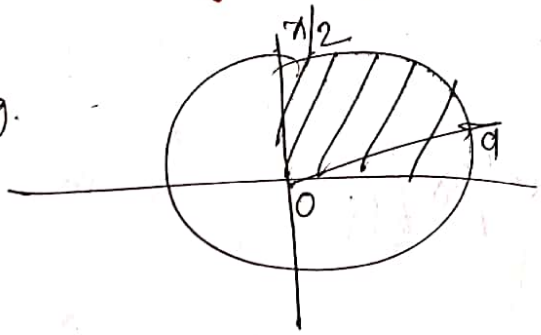
To change in polar form:

put $x = r\cos\theta$

$y = r\sin\theta$

$dx\,dy = r\,dr\,d\theta$

eg.



$\int_0^a \int_0^{\pi/2} r\cos\theta \cdot r\sin\theta \; r\,dr\,d\theta$

$= a^4/8$ //

Scanned by CamScanner

Q1. $\int_0^1 \frac{e^{\sqrt{1-x^2}}}{\sqrt{1-x^2}} x\,dx$

a) $\frac{e+1}{2}$ b) $\frac{e-1}{2}$ c) $e-1$ d) $e+1$

Q2. $f(x) = \int_x^0 \sin^2 t\,dt$ then $f'(x)$ is

a) 0 b) $\cos x^2$ c) $-\sin x^2$ d) $\sin 2x$

Q3. Value of Integration

$$\int_0^\infty \int_0^x x\, e^{-x^2/4}\,dy\,dx$$

a) 1 b) e c) 1/2 d) e/2

Q4. $I = \int_0^{\infty} \left\{ \int_{y/2}^{y/2+1} \left(\frac{2x-y}{2} \right) dx \right\} dy$

Substitute $u = \left(\frac{2x-y}{2}\right)$, $v = y/2$

then I is

a) $\int_0^4 \int_0^2 2u\,du\,dv$

b) $\int_0^4 \int_0^1 2u\,du\,dv$

c) $\int_0^4 \int_0^2 u\,du\,dv$

d) $\int_0^4 \int_0^1 u\,du\,dv$

Sol 1. $\sqrt{1+x^2} = t$

$1+x^2 = t^2$

$2x\,dx = 2t\,dt$

$dx = \frac{t\,dt}{\sqrt{t^2-1}}$

$$\int_1^0 \frac{e^t x\,dt}{t\sqrt{t^2-1}}$$

$$\Rightarrow -\int_0^1 \frac{e^t}{\sqrt{t^2-1}}$$

$$\Rightarrow \frac{1}{\sqrt{t^2-1}} \int_0^t e^t\,dx - \sin^{-1} e^t$$

$$-\int_0^1 \frac{e^t\,dt}{t\sqrt{t^2-1}}$$

Scanned by CamScanner

Sol 1 $\int_0^1 \frac{e^{\sqrt{1-x^2}}}{\sqrt{1-x^2}} x\,dx$

Sol $\sqrt{1-x^2} = t$

$1-x^2 = t^2$

$x^2 = 1-t^2$

$\frac{dt}{dx} = \frac{1}{2\sqrt{1-x^2}} \cdot (-2x)$

$dt = \frac{-x}{\sqrt{1-x^2}} dx$

$-\int_1^0 e^t\,dt$

$(e^t)_1^0 = -(1-e) = e-1$ //. C

Sol 2 $f(x) = \int_x^0 \sin t^2\,dt$ $f'(x)$ ü

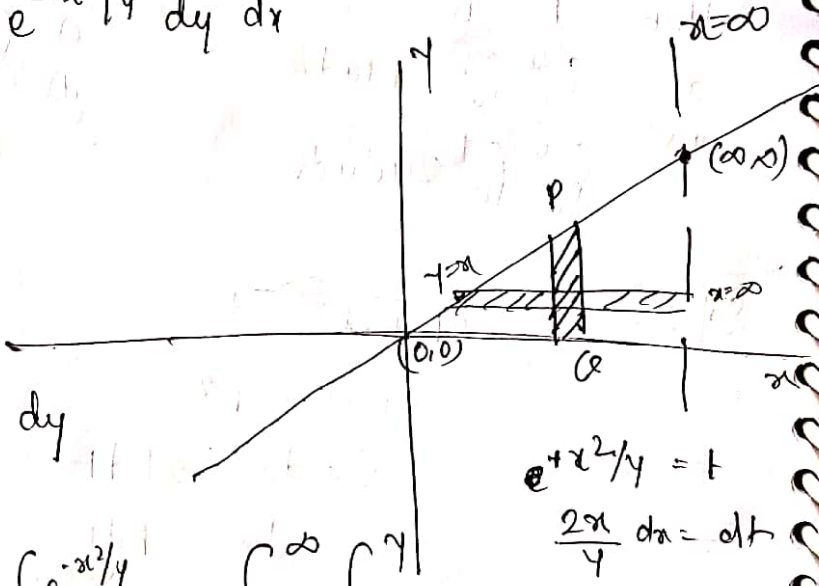
$f'(x) = 0 - \sin(x^2)$

$= -\sin x^2$ //.

Sol $\int_0^\infty \int_0^x x e^{-x^2/y}\,dy\,dx$

$y = 0$ | $x = 0$

$y = x$ | $x = \infty$



$\int_0^\infty \int_0^y x e^{-x^2/y}\,dx\,dy$

$e^{+x^2/y} = t$

$\frac{2x}{y}\,dx = dt$

$= \int_0^\infty \left[x \int e^{-x^2/y} - \int e^{-x^2/y} \right]$ $\int_0^\infty \int_0^y$

Scanned by CamScanner

$$\int_0^\infty \left[x \frac{e^{-x^2/4}}{-1/4} - \int \frac{e^{-x^2/4}}{-1/4} \right]$$

$$\Rightarrow \int_0^\infty \int_0^y \sqrt{4y}\, e^{-t} \frac{y}{2\sqrt{4y}}\, dt$$

$$\Rightarrow \int_0^\infty \int_0^{y/4} \left[\frac{e^{-t}}{2}\, dt \right]$$

$$\Rightarrow \int_0^\infty y \left(\frac{e^{-y}}{2} - \frac{1}{2} \right) dy$$

$$\Rightarrow \frac{1}{2} \left[y \frac{e^{-y}}{-1} - \frac{e^{-y}}{-1} \right]$$

$$\frac{1}{2} \left[-y e^{-y} - e^{-y} \right]_0^\infty$$

$$\Rightarrow \frac{1}{2} [1] \quad // \quad \text{Opt C} \quad //$$

If done this way

$$\int_0^x \int_0^\infty x e^{-x^2/4}\, dx\, dy$$

{ thinking of Gamma fun }

$$\frac{-x^2}{4} = t$$

$$-\frac{2x}{4}\, dx = dt$$

$$\int_0^x \int_0^\infty -\frac{e^{-t}}{2}\, y\, dt$$

$\Rightarrow -\frac{x^2}{2}$ which is wrong

a variable limit must be treated first

Q4. $$I = \int_0^8 \left\{ \int_{y/2}^{y/2+1} \left(\frac{2x-y}{2} \right) dx \right\} dy$$

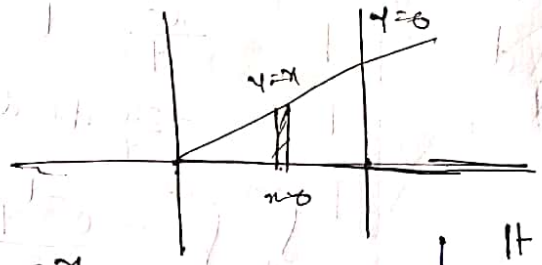
$$u = \frac{2x-y}{2}, \quad v = y/2$$

Scanned by CamScanner

Q. find the Value of $I = \iint_S x^2y^2\,dx\,dy$

where S is abounded region by the $y=x$, $x=1$ and $y=0$

Sol



$$\int_0^1 \int_0^x x^2y^2\,dy\,dx$$

$$\Rightarrow \int_0^1 x^2\left(\frac{y^3}{3}\right)_0^x dx$$

$$\int_0^1 x^2\left(\frac{x^3}{3}\right)$$

$$\Rightarrow \left[\frac{x^6}{18}\right]_0^1$$

$$\Rightarrow 1/18 \;//$$

It can also be Solved as

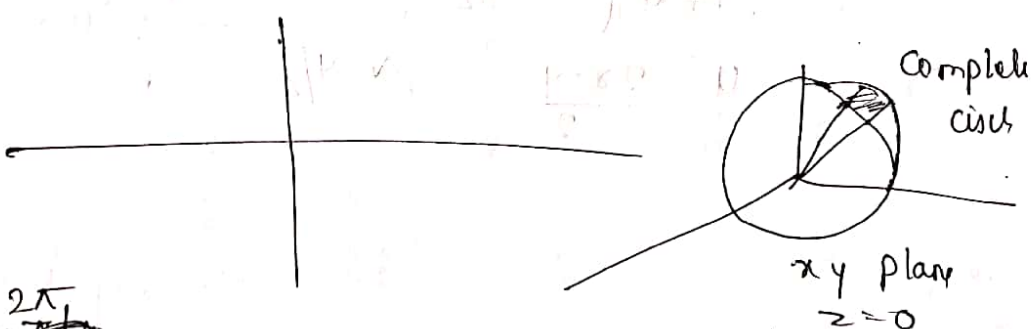
$$\int_0^1 \int_y^1 x^2y^2\,dy\,dx$$

$I = 1/18$ Same ans.

one benefit with first method
Both limit start from 0.
So calculation is less.
(So try for Vertical Strip for less Calculation)

Q. Same question where S is a boundary of the projection of sphere $x^2+y^2+z^2=1$ on xy plane

Sol



$$\int_0^1 \int_0^{2\pi} r^2\cos^2\theta\; r^2\sin^2\theta\; r\,dr\,d\theta$$

$$\Rightarrow \frac{1}{4}\int_0^1 r^5 (\sin 2\theta)^2\,dr\,d\theta =$$

Scanned by CamScanner

$$\Rightarrow \int_0^1 \int_0^{2\pi} r^2\cos^2\theta \; r^2\sin^2\theta \; r\,dr\,d\theta$$

$$\Rightarrow \int_0^1 r^5\,dr \int_0^{2\pi} \cos^2\theta \sin^2\theta \, d\theta$$

$$\frac{1}{4}\int_0^1 r^5\,dr \int_0^{2\pi} \sin^2 2\theta$$

$$\Rightarrow \frac{1}{4}\int_0^1 r^5\,dr \left(\frac{1-\cos 4\theta}{2}\right)$$

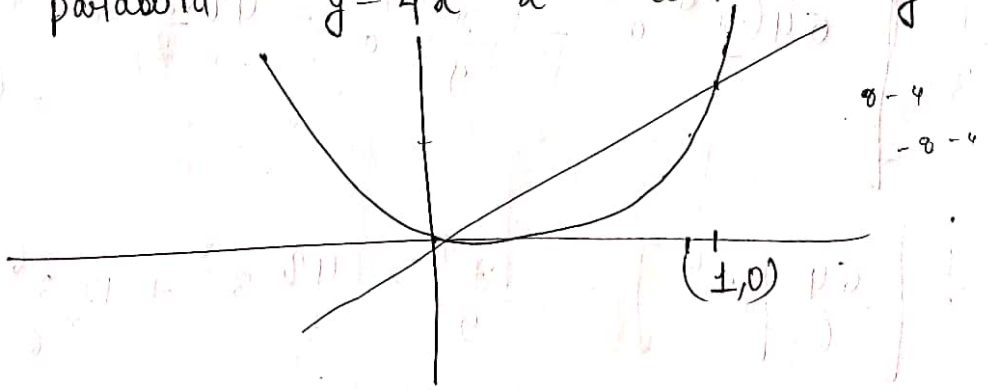
$$\frac{1}{4}\int_0^1 r^5\,dr \left[\frac{1}{2}(2\pi)\right] - \frac{1}{2\cdot 4}(\sin 4\theta)_0^{2\pi}$$

$$\frac{\pi}{4}\int r^5\,dr$$

$$\frac{\pi}{4}\left(\frac{r^6}{6}\right)_0^1$$

$$\frac{\pi}{4}\left(\frac{1}{6}\right) = \pi/24 \;//.$$

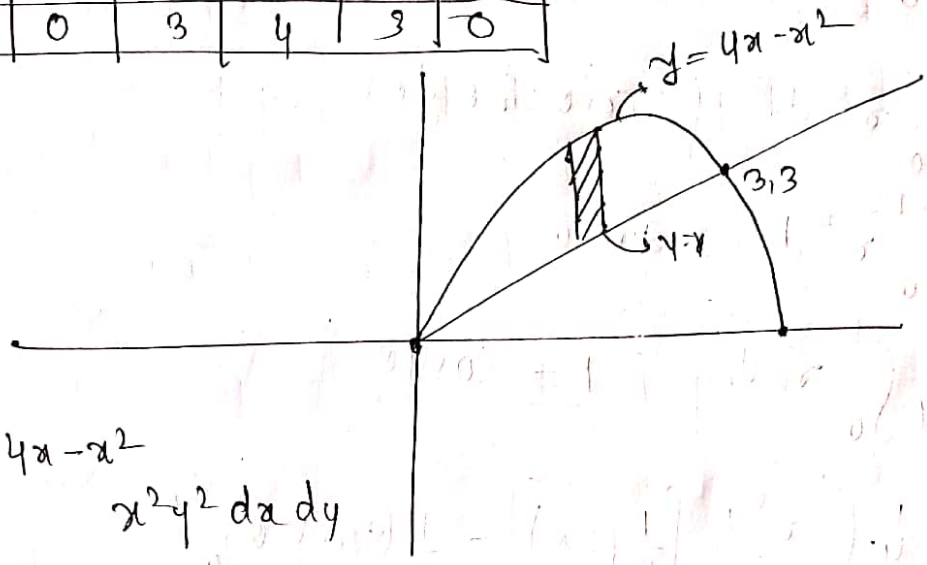
Q. Same question where S is region bounded by parabola $y = 4x - x^2$ and line $y = x$



Scanned by CamScanner

Sol $y = 4x - x^2$

x	0	1	2	3	4
y	0	3	4	3	0



$$\int_0^3 \int_x^{4x-x^2} x^2 y^2 \, dx \, dy$$

$$\Rightarrow \int_0^3 x^2 \left(\frac{y^3}{3}\right)_x^{4x-x^2} dx$$

$$\int_0^3 x^2 \left(\frac{(4x-x^2)^3}{3} - \frac{x^3}{3} \right)$$

$$\Rightarrow \int_0^3 x^2 \left[\frac{64x^3 - x^6 - 48x^4 + 12x^5 - x^3}{3} \right]$$

$$\Rightarrow \frac{1}{3} \left[64 \left(\frac{x^6}{6}\right)_0^3 - \left(\frac{x^9}{9}\right)_0^3 - 48 \left(\frac{x^7}{7}\right)_0^3 + 12 \left(\frac{x^8}{8}\right)_0^3 - \left(\frac{x^6}{6}\right)_0^3 \right]$$

$$\frac{1}{3} \left[\frac{64}{6} \left[3^6 \right] - \frac{3^9}{9} - \frac{48}{5} 3^5 + 12 \frac{3^6}{6} - \frac{3^4}{4} \right]$$

$$\frac{1}{3} \left[32 \cdot 3^5 - 3^7 - \frac{48}{5} 3^5 + 6 3^5 - \frac{3^4}{4} \right]$$

$$\Rightarrow$$

Scanned by CamScanner

$$3^4\left[32 - 3^2 - \frac{48}{5} + 6 - \frac{1}{12}\right]$$

$$3^4\left[32 - 9 + 6 - \frac{48}{5} - \frac{1}{12}\right]$$

$$3^4\left[29 - \frac{48}{5} - \frac{1}{12}\right]$$

$9.6 \quad 0.0833$

$$19.3167 \times 3^4$$

$$= 1564.65$$

Ans (Max) 104...

$$\frac{1}{3}\left[64\frac{3^6}{6} - \frac{3^9}{9} - 48\times\frac{3^7}{7} + 12\,\frac{3^8}{8} - \frac{3^6}{6}\right]$$

$$\frac{3^6}{3}\left[\frac{64}{6} - \frac{3^3}{9} - 48\frac{3}{7} + 12\cdot\frac{3^2}{8} - \frac{1}{6}\right]$$

$$3^5\left[\frac{64}{6} - 3 - 48\times\frac{3}{7} + \frac{3^3}{2} - \frac{1}{6}\right]$$

$$3^5\left[\quad\right]$$

Scanned by CamScanner

Green's Theorem

i) It gives the relationship between line and Surface Integral.

* ii) It is only valid for closed line Integral and open Surface Integral.

iii) It is a Scalar form of Stoke's theorem.

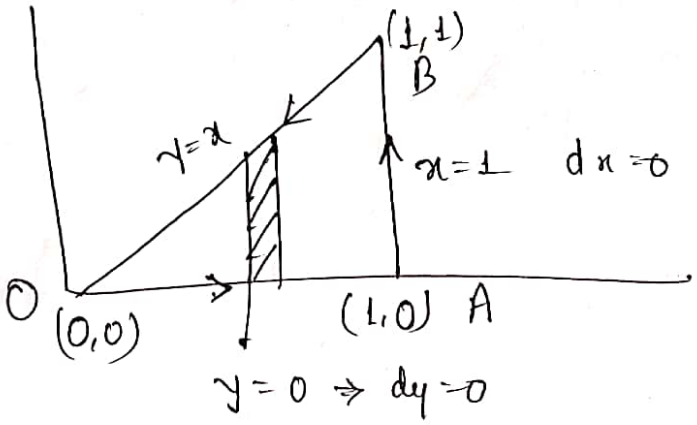
" If $\phi(x,y)$ & $\psi(x,y)$ are Continuous function such that $\frac{\partial \psi}{\partial x}$ & $\frac{\partial \phi}{\partial y}$ are exist in region R formed by closed curve C then according to green's theorem.

$$\int_C \phi\,dx + \psi\,dy = \iint_R \left(\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right) dx\,dy$$

note:- for Surface Integration if we take Vertical Strip then line Integral Should be anticlockwise.

Q. Find the Value of $\oint_C x^2y\,dx + x^2dy$ where C is a triangle with Vertices (0,0) (1,0) (1,1)

Sol)



Scanned by CamScanner

$$\oint_C \phi\,dx + \varphi\,dy = \iint_R \left(\frac{\partial \varphi}{\partial x} - \frac{\partial \phi}{\partial y}\right) dx\,dy$$

Take L.H.S

Line Integral

along OA $y = 0$, $dy = 0$

$$I_{OA} = \int_0^1 0\,dx = 0$$

along AB:

$x = 1$ $dx = 0$

$$I_{AB} = \int_0^1 0 + 1^2\,dy$$

$$= 1$$

along BO:- $y = x$

$$I_{BO} = \int_1^0 x^3\,dx + x^2\,dx$$

$$= \int_1^0 (x^3 + x^2)\,dx$$

$$= \left(\frac{x^4}{4} + \frac{x^3}{3}\right)_1^0$$

$$= -\frac{1}{4} - \frac{1}{3}$$

$$= -\frac{7}{12}$$

$$I_{OA} + I_{BO} + I_{AB} = 1 - \frac{7}{12}$$

$$= 5/12 \quad //$$

Scanned by CamScanner

By Greens theorm

$$\int_0^1 \int_0^x (2x - x^2)\, dy\, dx$$

$$\Rightarrow \int_0^1 2x^2 - x^3\, dx$$

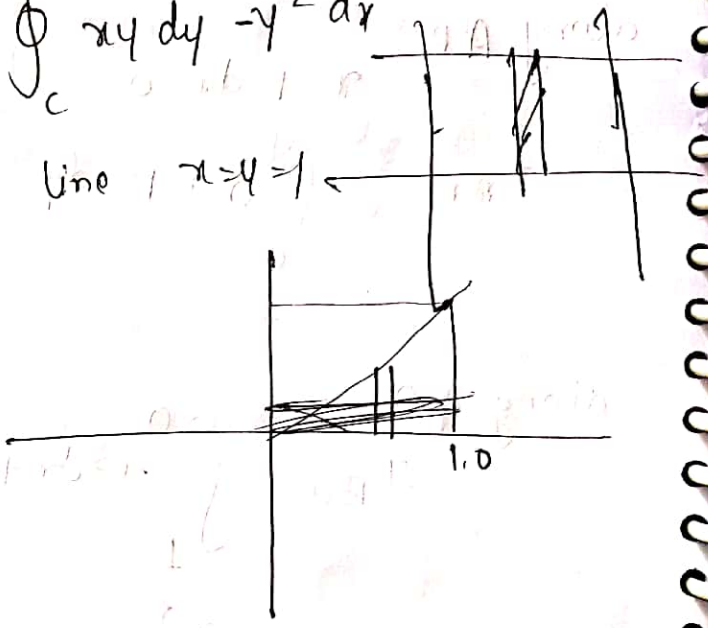
$$5/12 \quad //$$

$\phi = x^2 y \quad \frac{\partial \phi}{\partial y} = x^2$

$\psi = x^2 \quad \frac{\partial \psi}{\partial x} = 2x$

Q. Value of Integral $\oint_C xy\, dy - y^2\, dx$

when C: Square Cut by line $x=y=1$



Sol

$\phi = xy$

$\phi = -y^2$

$$\int_0^1 \int_0^x$$

$$\int_0^1 \int_0^x y + \frac{y^3}{3}$$

$$\frac{1}{3}\left[3\frac{x^2}{2} + \frac{x^4}{4} \right]_0^1$$

$$\frac{1}{3}\left[3\frac{1}{2} + \frac{1}{4} \right]$$

$$\frac{1}{2} + \frac{1}{12} \qquad \frac{6+1}{12}$$

$\frac{\partial \psi}{\partial x} = y$

$\frac{\partial \phi}{\partial y} = -y^3$

$(3/2)$

Scanned by CamScanner

Q. find the Value of integral

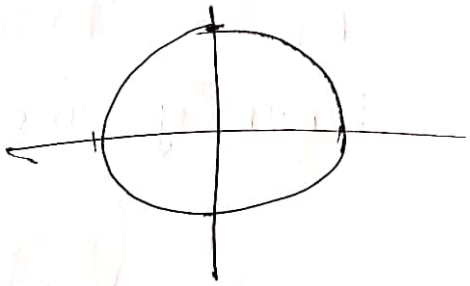
$$\oint_C x\,dy - y\,dx$$ along closed path $x^2 + y^2 = 1$

a) π b) 2π c) 4π d) 8π

Sol

$\phi = -y$ $\quad \frac{\partial \phi}{\partial y} = -1$

$\psi = x$ $\quad \frac{\partial \psi}{\partial x} = 1$



$$\int_0^1 \int_0^{2\pi} (1 + 1)\, dy\, dx$$

$$\int_0^1 r^2 \int_0^{2\pi} \sin 2\theta \, d\theta$$

$$\int_0^1 r^2 - \left(\frac{\cos 2\theta}{2}\right)_0^{2\pi}$$

Scanned by CamScanner

Applications (Area – Length – Volume)

① Area $= \iint_S dx\,dy$

[for limit : Trace S]

② Length of Curve $y = f(x)$

$$L = \int_c \left\{\sqrt{1+\left(\frac{dy}{dx}\right)^2}\right\} dx$$

for limit trace the Curve.

Volume : $y = f(x)$

(A) Volume formed by revolving the Curve

a) Revolved by X axis

$$V_x = \int_a^b \pi y^2 dx$$

b) Revolved by Y axis

$$V_y = \int_a^b \pi x^2 dy$$

(B) Volume : (3D)

$$V = \iiint_{V} dx\,dy\,dz$$

$\uparrow_z$

Scanned by CamScanner

$$V = \iint_{S \to xy} z\,dx\,dy \quad \left\{ \text{Volume as double Integral.} \right.$$

$z \to f(x,y)$

Case $\iint_{S \to xz} y\,dx\,dz$ $\quad y \to f(x,z)$ given

$$V = \iint_S f(x,z)\,dx\,dz$$

Q1. Find the area of region bounded by $y^2 = 4ax$ and its Latus rectum in first quadrant.

Q2. Find length of Curve

$$y = \frac{2}{3}x^{3/2} \quad ; \; x \text{ varies from } 0 \text{ to } 1$$

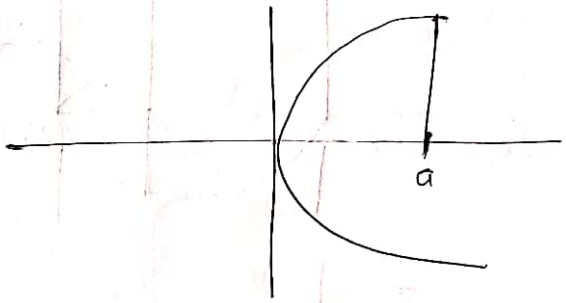
a) 1.22 b) 2.22 c) 3.22 d) 4.22

Q3. Find Vol. formed by cylinder $x^2+y^2=4$ & plan $y+z=4$ & $z=0$

a) 4π b) 8π c) 16π d) 32π

Sol 1:-

$$\int_0^a \int_0^{\sqrt{4ax}} dx\,dy$$



$$\int_0^a \sqrt{4ax}\,dy$$

$$\Rightarrow \frac{2}{3}\sqrt{4a}\cdot x^{1/2+1} \qquad \frac{2}{3}\sqrt{4a}\; a^{3/2} \Rightarrow \frac{4}{3}\cdot a^2$$

Scanned by CamScanner

Sol 2

$$L = \int_C \left\{ \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \right\} dx$$

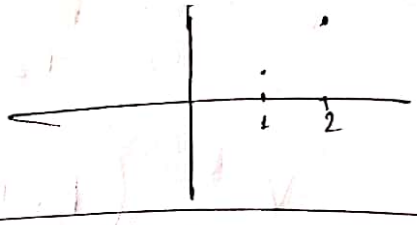
$$\int_0^{2/3} \int_0^1 \sqrt{1 + (x^{5/2})^2}\, dx$$

$$\int_0^{2/3} \int_0^1 \sqrt{1 + x^5}\, dx$$

$$y = \frac{2}{3} x^{3/2}$$

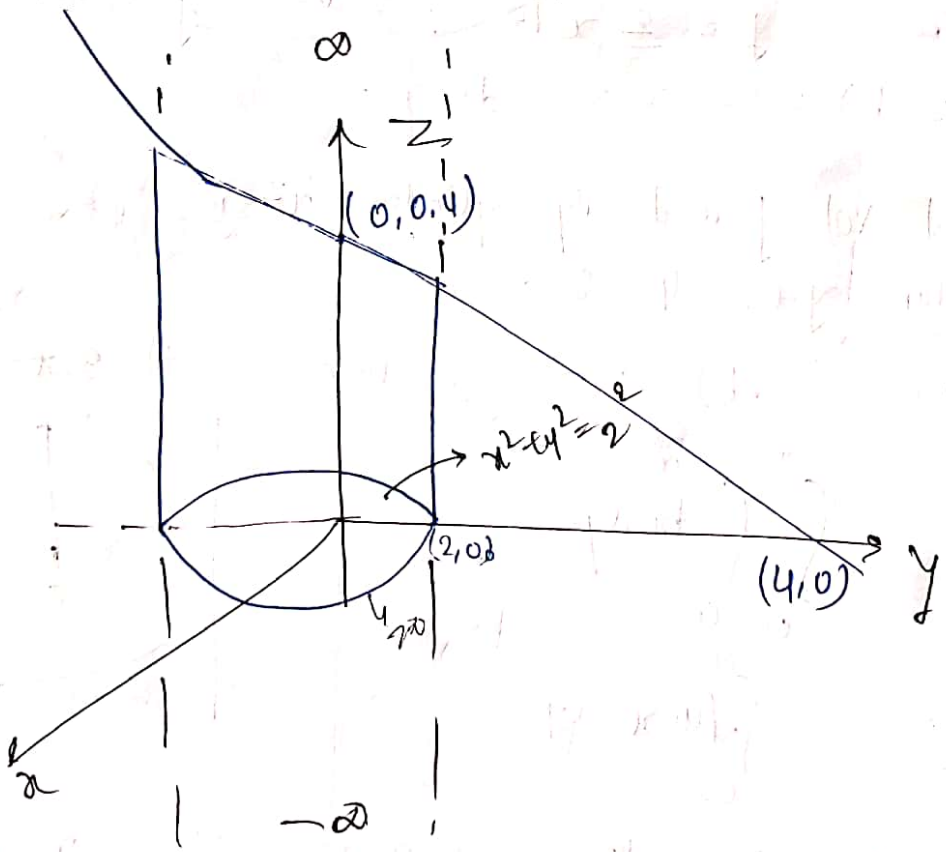
$$y^2 = \frac{2}{3} x^3$$

x	0	1	2	3
y	0	2/3	5	



x	
y	

Q3.
Sol



Scanned by CamScanner

$$V = \iint_S z \, dx \, dy$$

$$= \iint_{S \longrightarrow \text{circle}} (4 - y) \, dx \, dy$$

$$\int_0^2 \int_0^{2\pi} (4 - r\sin\theta) \, r \, dr \, d\theta$$

$$\int_0^2 \int_0^{2\pi} 4r \, dr d\theta - r^2 \sin\theta \, dr \, d\theta$$

$$\int_0^2 dr \left(2\pi \cdot 4r + r^2 \cos\theta \right)$$

$$\int_0^2 8\pi r + r^2 \, dr$$

$$\left[8\pi \frac{r^2}{2} + \frac{r^3}{3} \right]_0^2 = 8\pi \frac{4}{2} + \frac{8}{3}$$

$$16\pi$$

$$\int_0^2 4r(2\pi)$$

Scanned by CamScanner

Q1 find the Volume bounded by $y = x^2$, $x = y^2$, & planes $z = 0$ & $z = 3$

a) 1 b) 2 c) 3 d) 4

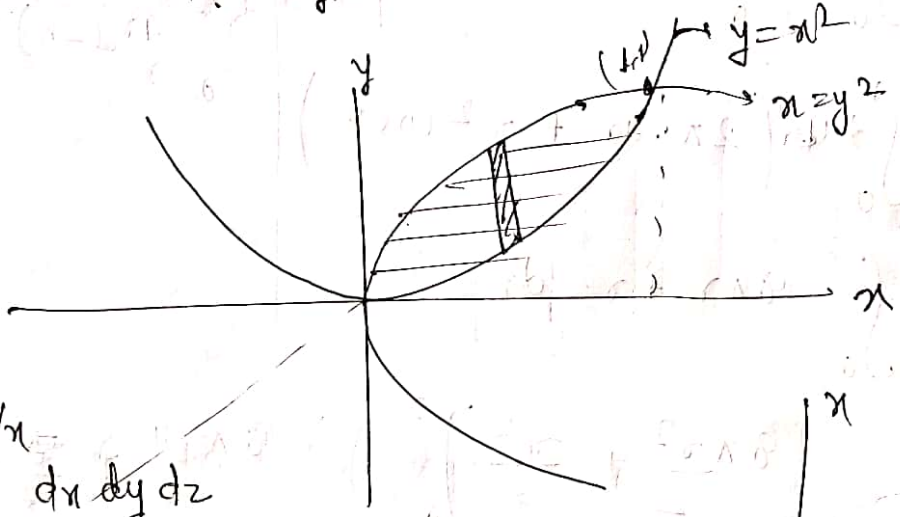
Q2 find the Vol. generated by revolving the area bounded by parabola $y^2 = 8x$ & line $x = 2$, about x axis

a) 4π b) 8π c) 12π d) 16π

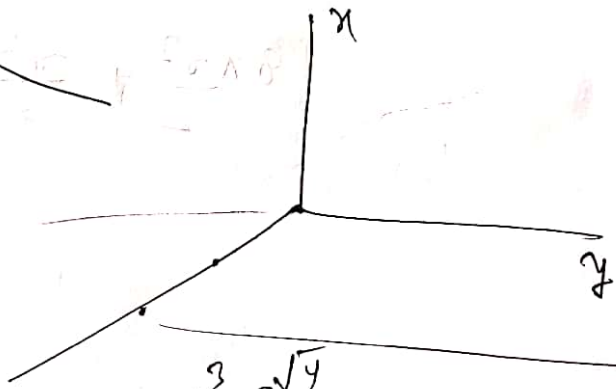
Q3. find the Integral $\oint_C (x^2 + y^2)\,dx - 2xy\,dy$

where C is closed path bounded by parabola $y = x^2$ and line $y = x$

Sol 1.



$$\int_0^3 \int_0^{\sqrt{y}} \int_0^{\sqrt{x}} dx\,dy\,dz$$



$$3 \int_0^{\sqrt{y}} \sqrt{x}\,dx$$

$$3 \cdot \frac{2}{3}\left(x^{3/2}\right)_0^{\sqrt{y}}$$

$$= 2$$

$$\cdot \sqrt{x}$$

$$\left[\frac{2}{3} x^{3/2}\right]_0^{y}$$

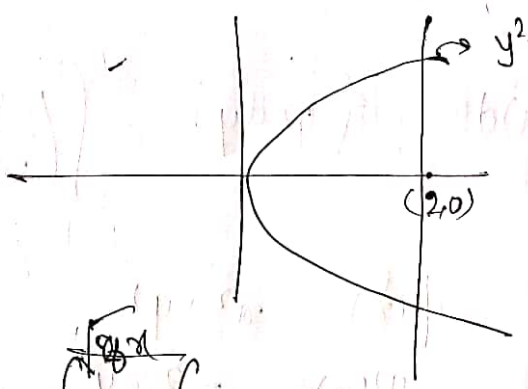
$$\int_0^3 \int_0^{\sqrt{y}} \sqrt{x}\,dx\,dz$$

$$\int_0^3 \left(\frac{x^{3/2}}{2/3}\right)_0^{\sqrt{y}}$$

$$\int_0^3 \frac{3}{2} y^3 = \frac{3}{2 \cdot 4} 3^4$$

Scanned by CamScanner

Sol 2



$V_x =$ ~~$V_x = \int_{-\sqrt{8x}}^{\sqrt{8x}} \pi y^2\,dx$~~ ⟹ $\pi \int_0^2 8x\,dx$

$= 8x2\pi = 16\pi$ //.

$= \int_{-\sqrt{8x}}^{\sqrt{8x}}$

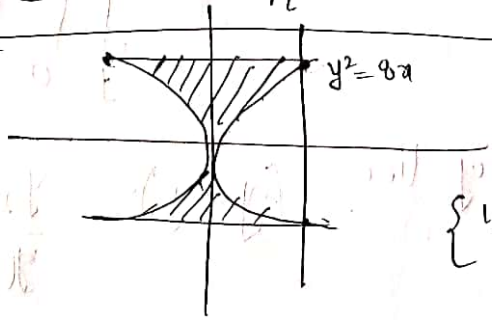
Sol 1

$$\int_0^3 \int_0^1 \int_{x^2}^{\sqrt{x}} dy\,dx \Rightarrow 3\int_0^1 \left(\sqrt{x} - x^2\right) dx$$

$$= 3\left[\frac{x^{3/2}}{3/2} - x^{3/3}\right]$$

$$3\left[\frac{2}{3} - \frac{1}{3}\right] = 1$$ //.

Sol 2 Revolve about y axis



{1 to 2}

$$V_y \int_{-4}^{4} \pi \frac{y^4}{64}\,dy$$

$$= 2\int_0^4 \frac{y^5 \pi}{5 \times 64} \qquad \frac{4^5 \times 2}{5 \times 4^3} = \frac{4^2 \times 2}{5}\pi = \frac{32\pi}{5}$$

V_y (with line) $\int_0^2 \int_{-\sqrt{8x}}^{\sqrt{8x}}$

Scanned by CamScanner

Sol3

$$\int \phi(x)dx + \oint \psi\, dy = \int\left(\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right)dx\,dy$$

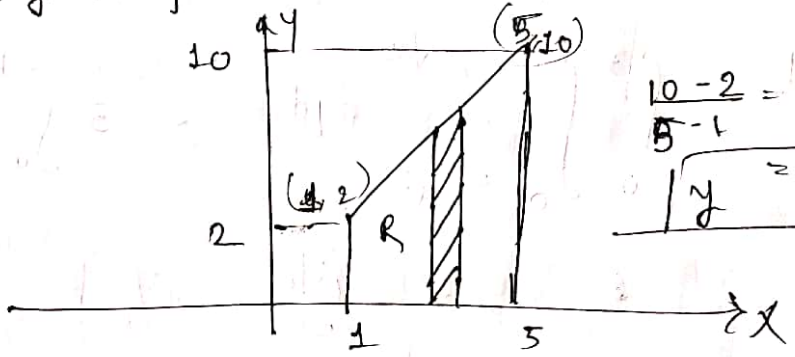
$$\phi(x) = x^2 + y^2$$

$$\psi(x) = -2xy$$

⇒

2017

Q. $I = c\int\int_R xy^2\,dx\,dy$; $c = 6\times10^{-4}$



$$\frac{10-2}{5-1} = \frac{+8}{4}$$

$$y = 2x$$

Sol $I = 6\times10^{-4}\int_1^5\int_0^{2x} xy^2\,dx\,dy$

eqn of line $(y-y_1) = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$$y = 2x$$

$$I = 6\times10^{-4}\int_1^5\int_0^{2x} xy^2\,dx\,dy \Rightarrow \int_0^5 x\,\frac{x^3}{3}$$

$$= \left(\frac{5^5}{15} - \frac{1}{15}\right)6\times10^{-4} = \left(\frac{x5}{15}\right)_1^5$$

Scanned by CamScanner

* Improper Integral

1st kind → for

$$I = \int_a^b f(x)\,dx \quad \cdots (1)$$

If $a = -\infty$ or $b = +\infty$ or both or any one is ∞ but $f(x)$ is finite in closed Interval $[a,b]$ then eqn (1) is said to be Improper Integral of first kind.

(i) $I = \int_{-\infty}^{\infty} e^{-x^2}\,dx$

$= e^{-(\infty)^2} - e^{-(-\infty)^2}$

= finite

Improper

(ii) $I = \int_0^{\infty} e^{-x}\,dx$

= finite

Improper

(iii) $I = \int_{-\infty}^{\infty} e^{-x}\,dx$

$= \infty$ not Improper

(iv) $\int_0^{\infty} \frac{1}{x-1}\,dx$

for $x = 0$ I = finite

for $x = \infty$ I = finite

$x \in [0, \infty)$ but not for $x = 1$

So, Not Improper.

2nd kind :

$$I = \int_a^b f(x)\,dx$$

If a or b is finite and $f(x)$ is infinite in closed $[a,b]$

ex (1) $I = \int_2^4 \frac{1}{x-1}\,dx$

$f(x)$ is finite $[2,4]$ except $x = ?$

$f(3) = \infty$

So Improper

Scanned by CamScanner

eg (2) $I = \int_2^4 \frac{[x^2+1]}{(x-3)(x-2)} dx$

$x = 2 \quad f(x) \to \infty$

Improper

Convergence of Improper Integral

1) If Improper Integral $\int_a^b f(x)\,dx$ is finite Value then Integral is Said to be Convergent

2) If Improper Integral $\int_a^b f(x)\,dx$ = Infinite $\infty / -\infty$

Then Integral is Said to be divergent.

Q1 $\lim_{a\to\infty} \int_1^a x^{-4} dx$ a) Divergent b) converge to $1/3$ c) Converge to $-1/\infty$ d) Converge to 10

Q2. $\int_{-\infty}^{0} \frac{\sinh(x)}{\frac{e^x - e^{-x}}{2}}\, dx$ is a) Convergent b) Divergent

Q3. $\int_{-1}^{1} \frac{1}{x^2} dx$ is ___ $\Rightarrow 2\int_0^1 \frac{1}{x^2} = -2\left[\frac{1}{x}\right]_0^1$

$-2 + \infty = \infty$

Divergent

Sol 1 $\int_1^\infty x^{-4} dx = \left[\frac{x^{-3}}{-3}\right]_1^\infty = -\frac{1}{3}\left[\frac{1}{\infty} - 1\right]$

$= 1/3$

Converge to $1/3$

Scanned by CamScanner

So|2 $\int_{-\infty}^{0} \sinh x \, dx \Rightarrow \left[\cosh x\right]_{-\infty}^{0}$

$\Rightarrow \left[\frac{e^{x}+e^{-x}}{2}\right]_{-\infty}^{0}$

$\Rightarrow \infty$ divergent

note: for function $y = f(x)$ the value of x for which the function has finite real value is known as the domain of the function. but for differentiation or integration no need to check the real value only check whether function is defined with finite value in the given limit or not.

$f(x) = \frac{\sqrt{1-x}}{\sqrt{1+x}}$

$= x \neq -1$ and $x < 1$

if $x = 2$ or more, imaginary

Scanned by CamScanner